Generalized linear models of neural activity

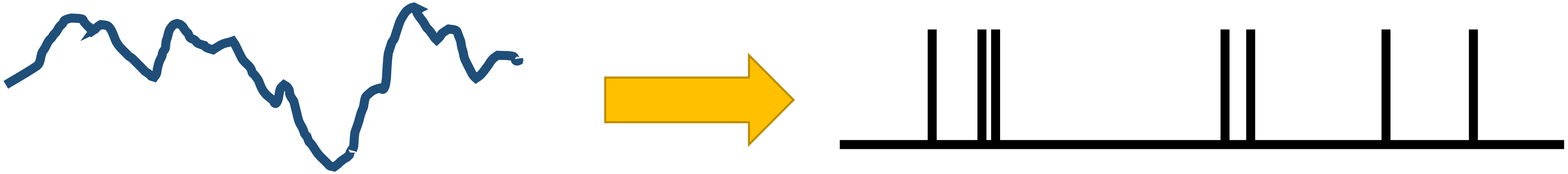
Bi23: Methods in Neural Data Analysis

2/1/2019

Neural encoding

Stimulus feature

Recorded spikes



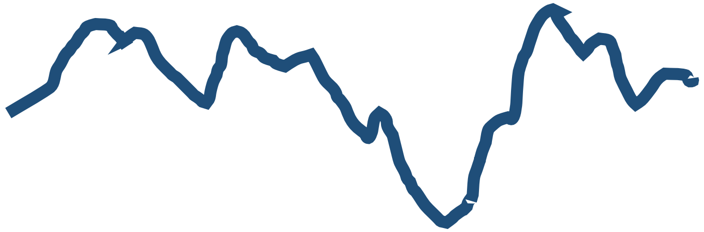
time →

time →

- Brightness of a screen
- Concentration of an odor
- Proximity of a predator
- Movement of a limb

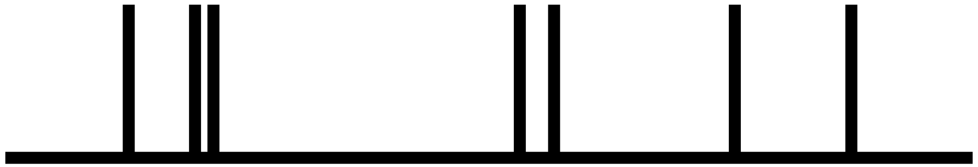
Neural encoding

Stimulus feature **(x)**



time →

Recorded spikes **(y)**



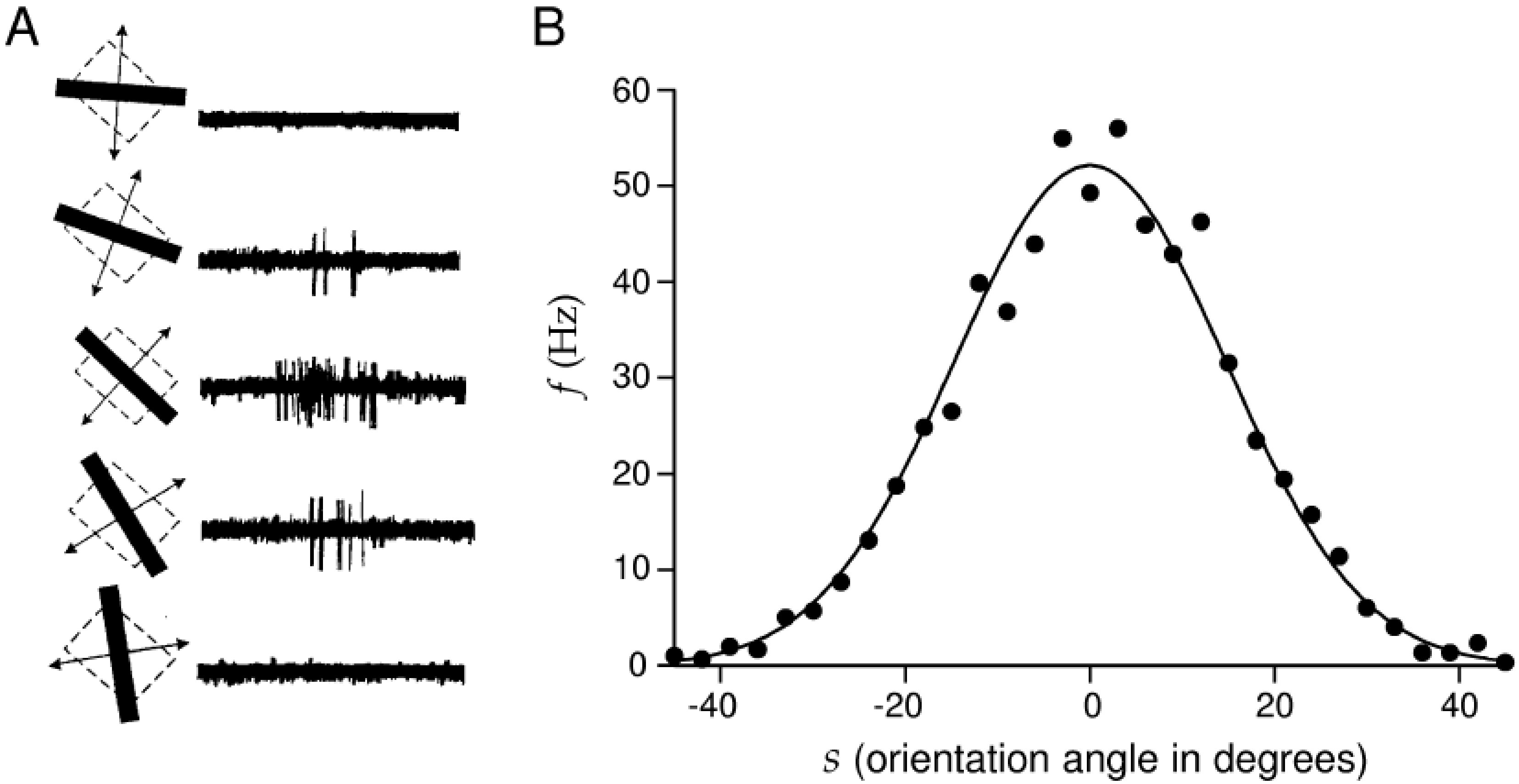
time →

Encoding model

P (y | x)

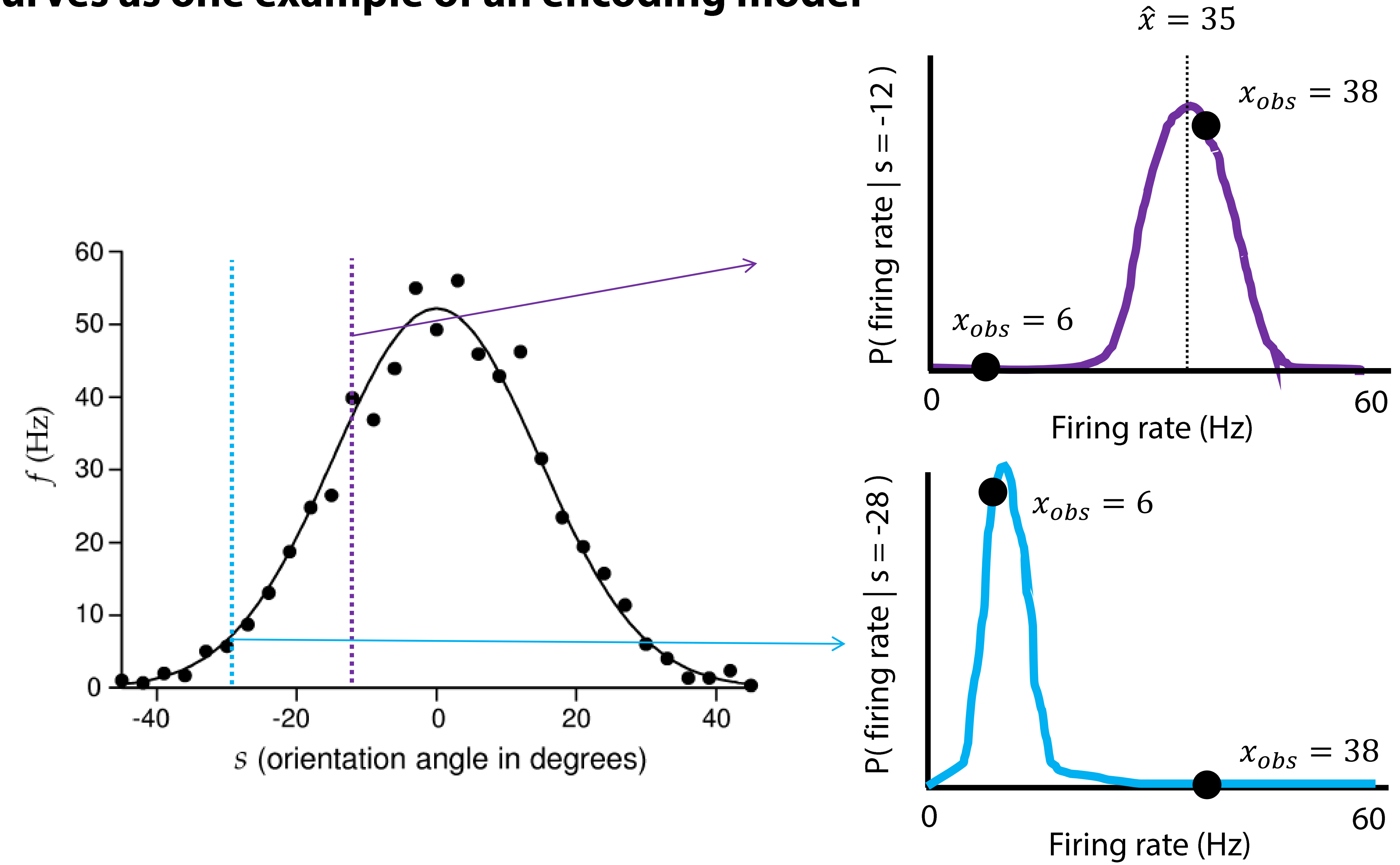
Given that the animal saw stimulus "x", what pattern of spiking "y" do we expect to observe in its brain?

Tuning curves as one example of an encoding model



Gaussian tuning curve of a cortical (V1) neuron

Tuning curves as one example of an encoding model



An example "Gaussian" neuron model

Underlying spike rate

$$\mu = \theta x$$

unknown parameter

stimulus

Predicted spike distribution

$$y \sim N(\mu, \sigma^2)$$

An example "Gaussian" neuron model

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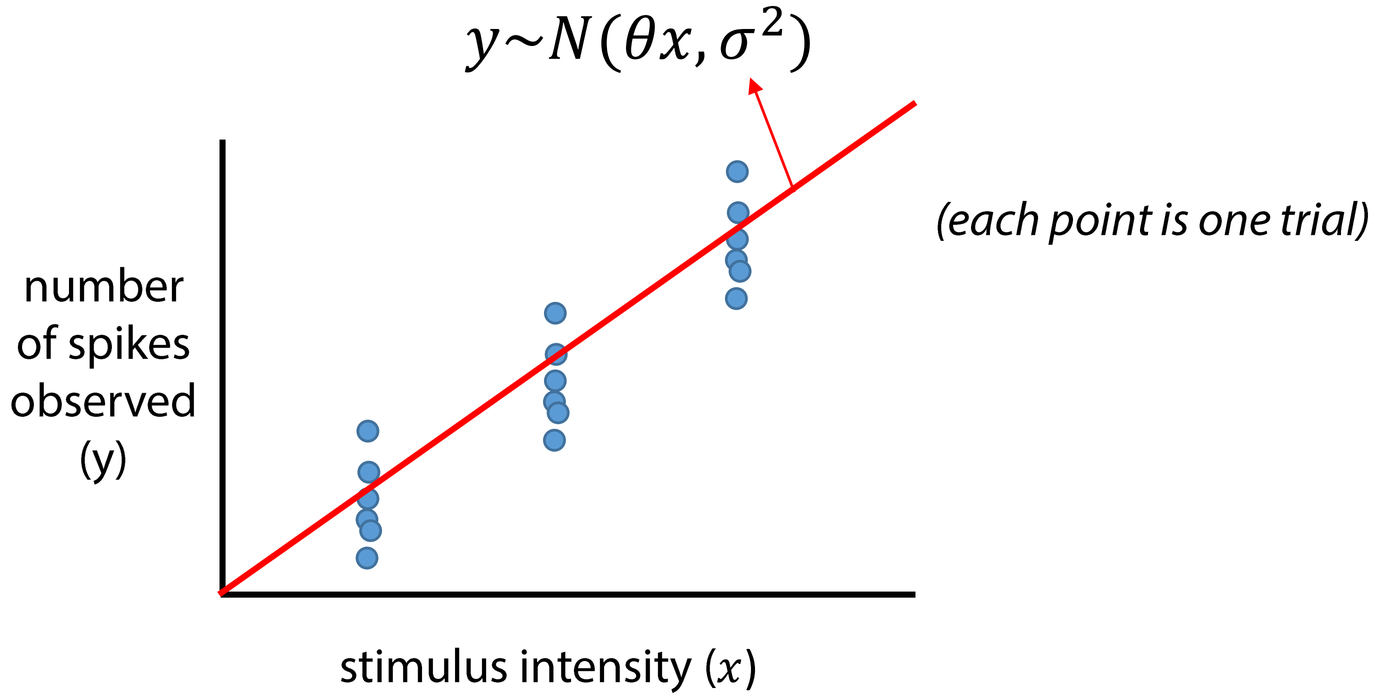

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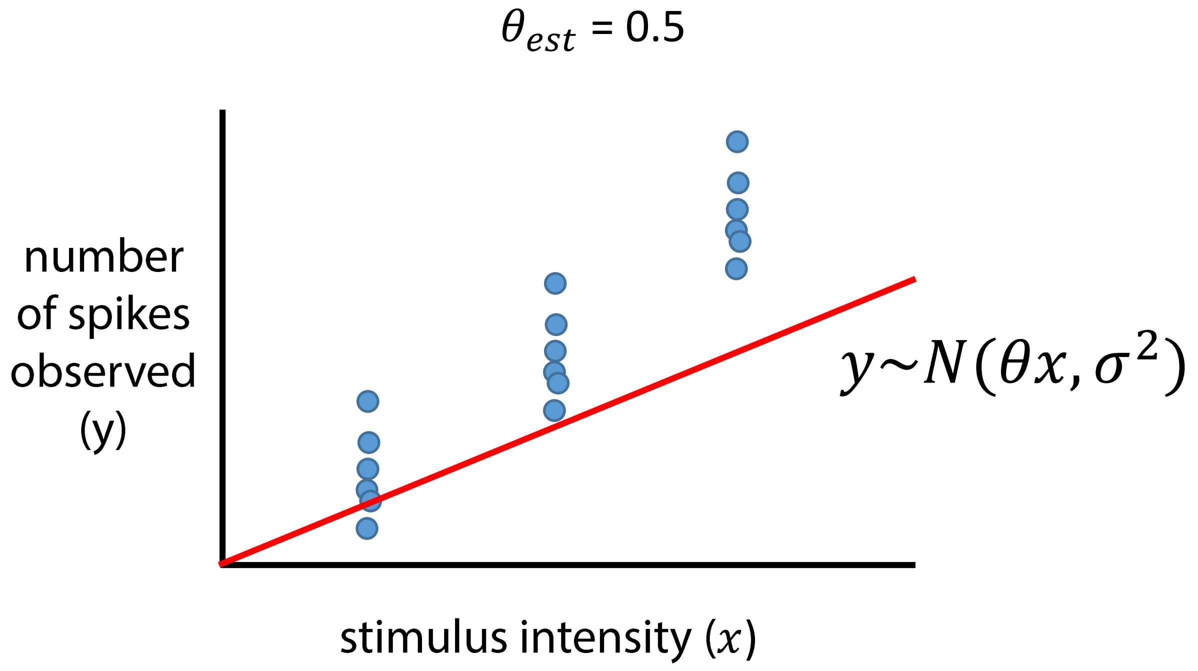

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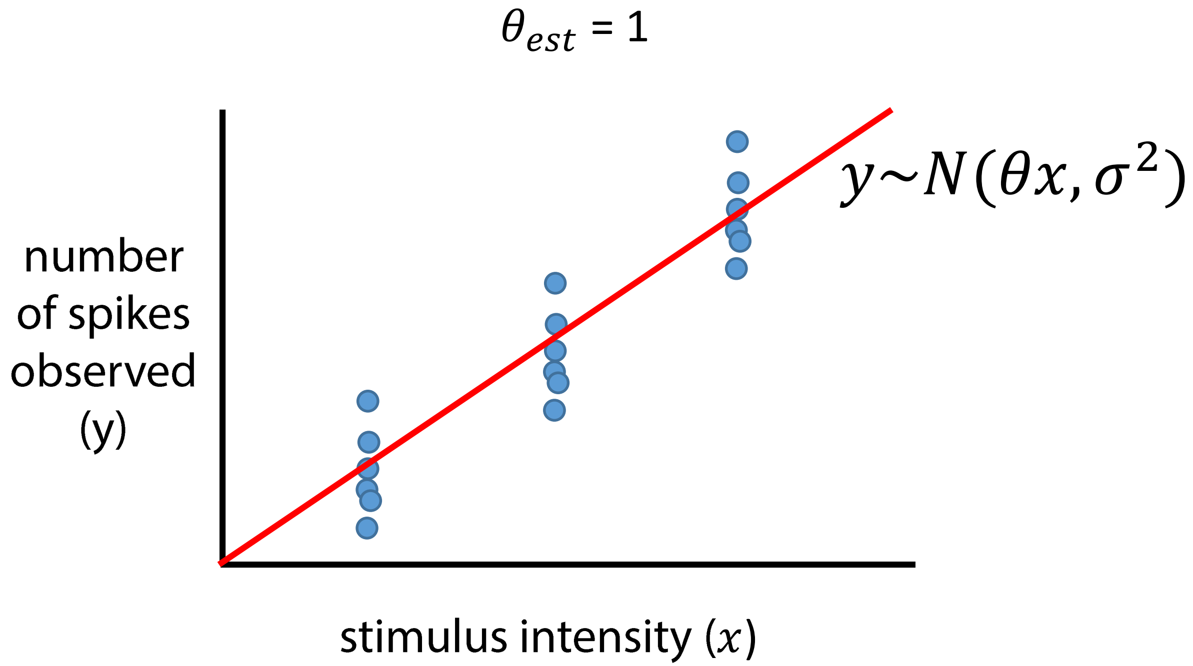


An example "Gaussian" neuron model

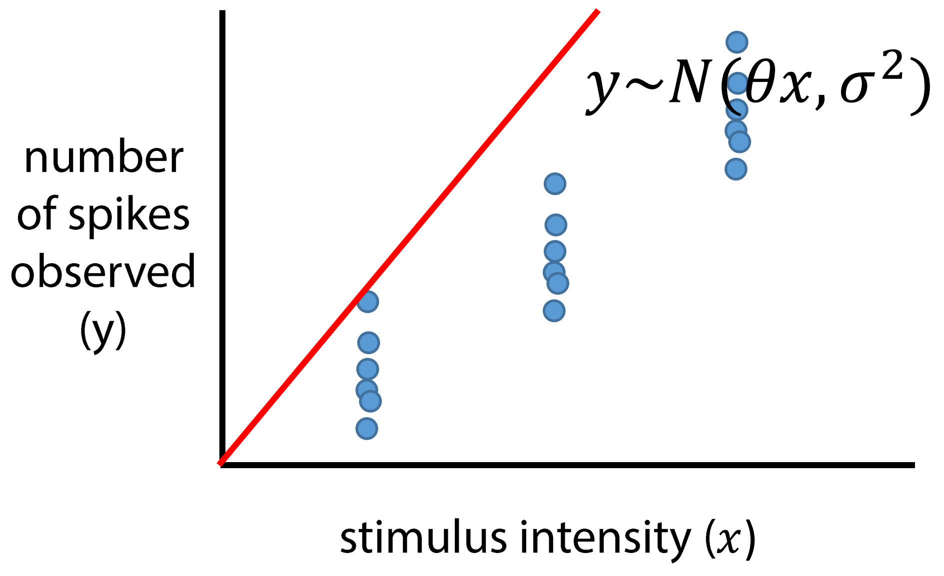
Predicted spike distribution $y \sim N(\mu, \sigma^2)$

Underlying Spike rate $\mu = \theta x$

unknown parameter

stimulus

θ_{est} = 1.5



Fitting the Gaussian model

Predicted spike distribution

$$y \sim N(\mu, \sigma^2)$$

$$P(y|x, \theta, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(y-\theta x)^2}{2\sigma^2}}$$

Underlying Spike rate

$$\mu = \theta x$$

unknown parameter

stimulus

Now, find the "most likely" θ given observed y and x!

Maximum likelihood (ML) estimation of θ:

$$P(\mathbf{y}|\mathbf{x},\theta,\sigma) = \prod \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(y_i-\theta x_i)^2}{2\sigma^2}}$$

We want to find θ that maximizes P(y|x, θ) for all our observed y and x; we'll assume the trials are independent of each other.

$$\log\left(P(\mathbf{y}|\mathbf{x},\theta,\sigma)\right) = \frac{-\sum (y_i - \theta x_i)^2}{2\sigma^2} + c$$

Any θ that maximizes P(y|x, θ) also maximizes its log

$$\frac{d}{d\theta}\log\left(P(\mathbf{y}|\mathbf{x},\theta,\sigma)\right) = -\sum \frac{(y_i - \theta x_i)x_i}{\sigma^2} = 0$$

The maximum of log(P(y|x,θ) occurs where its derivative is zero.

$$\hat{\theta} = \frac{\sum y_i x_i}{\sum x_i^2}$$

This is our *maximum likelihood estimate* of θ!

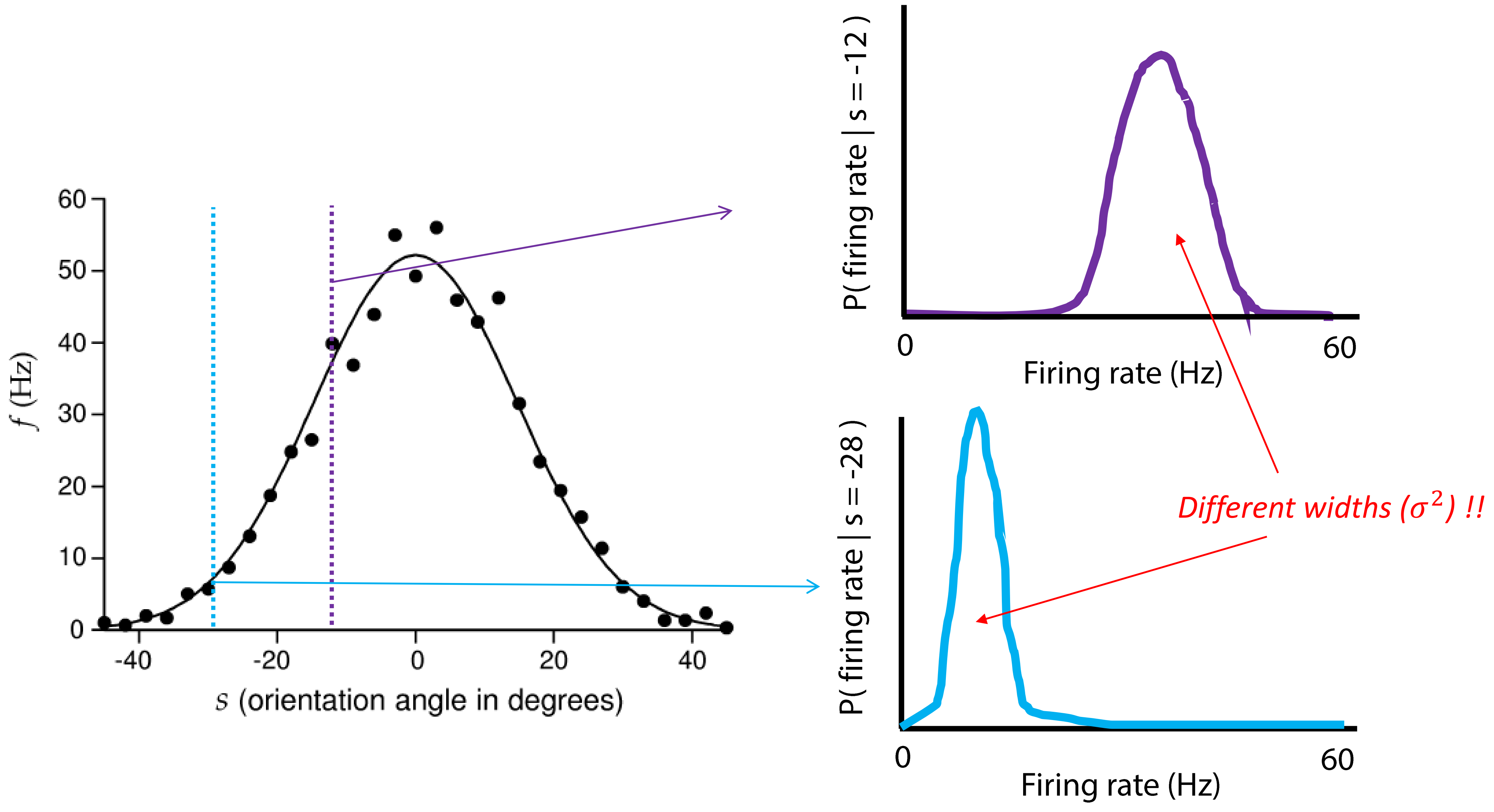
Maximum likelihood estimation

- Has an analytical-form solution for θ provided you use a certain class of models ("exponential family" models)

- Your estimate $\hat{\theta}$ converges to the actual ground-truth θ if you have enough data (ie there's no *bias*)

- Is the most efficient (minimum-error) unbiased way to estimate model parameters

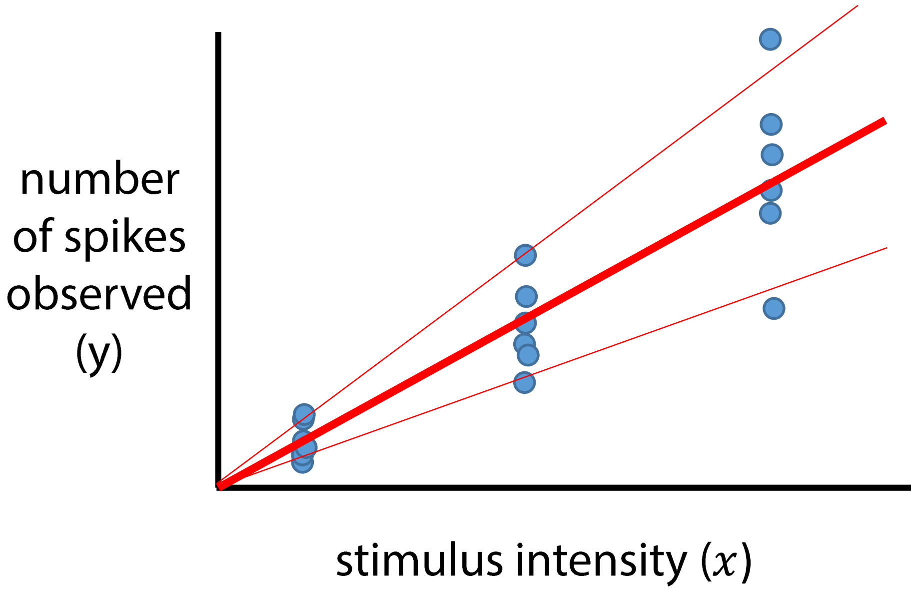
We don't have to assume a Gaussian model



Another model class that works well for neural data: Poisson models

Predicted spike distribution $y \sim Poiss(\lambda)$

Underlying spike rate $\lambda = \theta x$



Encoding model $$P(y|x,\theta) = \frac{1}{y!}(\theta x)^y e^{-(\theta x)}$$

$$\text{mean}(y) = \lambda = \theta x$$

$$\text{var}(y) = \lambda = \theta x$$

We can solve for the ML estimate of θ for the Poisson model, too

Encoding model

$$P(\boldsymbol{y}|\boldsymbol{x},\theta) = \prod \frac{1}{y_i!}(\theta x_i)^{y_i} e^{-(\theta x_i)}$$

$$\log\big(P(\mathbf{y}|\mathbf{x},\theta)\big) = \log(\theta)\sum y_i - \theta \sum x_i + c$$

$$\frac{d}{d\theta}\log\big(P(\mathbf{y}|\mathbf{x},\theta)\big) = \frac{1}{\theta}\sum y_i - \sum x_i \quad = \quad 0$$

$$\boxed{\hat{\theta} = \frac{\sum y_i}{\sum x_i}}$$

Taking a step back

Encoding model

$$P(y|x,\theta)$$

stimulus "x"
spiking "y"
parameter(s) "θ"

For a given y: an expected probability distribution of spike counts

For a given θ: the probability of seeing spiking y given our model parameters

For a given x: the "stimulus likelihood function"- the stimulus for which the observed spikes are most probable

Now: Generalized Linear Models

Poisson encoding model

$$P(y|\lambda) = \frac{1}{y!}\lambda^y e^{-\lambda}$$

Underlying spike rate

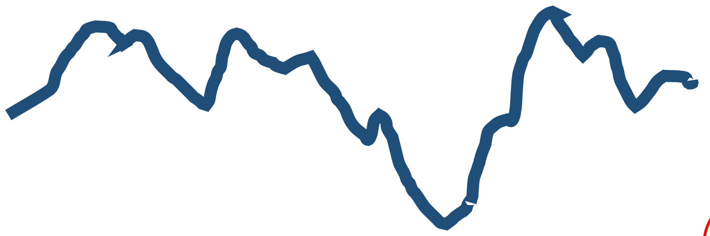
$$\lambda = \theta x$$

What goes into our spike rate λ?

Short answer: whatever we want

Now: Generalized Linear Models

Stimulus feature **(x)**



time →

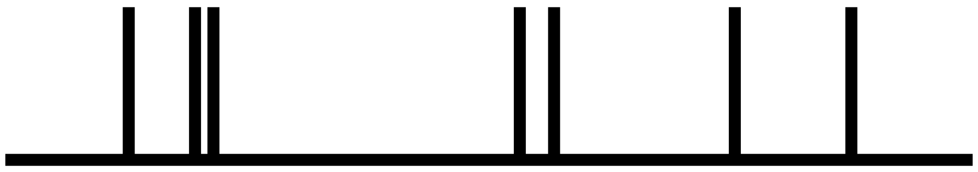
Encoding model

$$P(y_t|\lambda) = \frac{1}{y_t!}\lambda^{y_t}e^{-\lambda}$$

Underlying spike rate

$$\lambda = f(x_{1:t}, y_{1:t-1})$$

Recorded spikes **(y)**



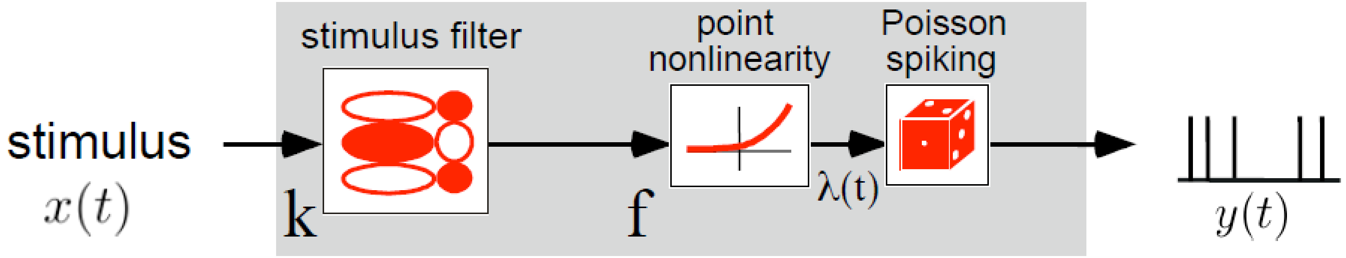
time →

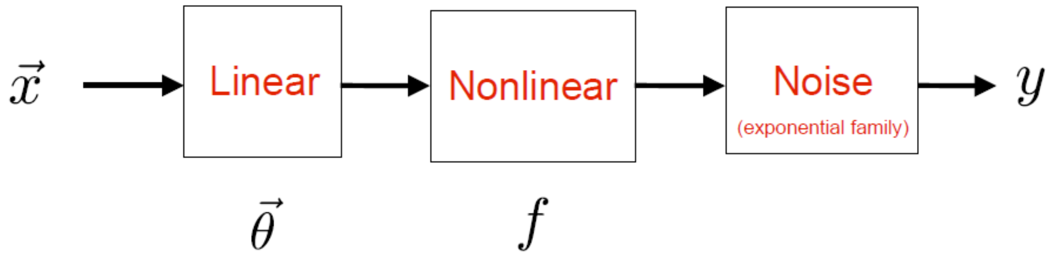
Now: Generalized Linear Models

Stimulus encoding model

$$P(y_t|\lambda) = \frac{1}{y_t!}\lambda^{y_t}e^{-\lambda}$$

$$\lambda = f(x_{1:t}, y_{1:t-1}, \ldots)$$



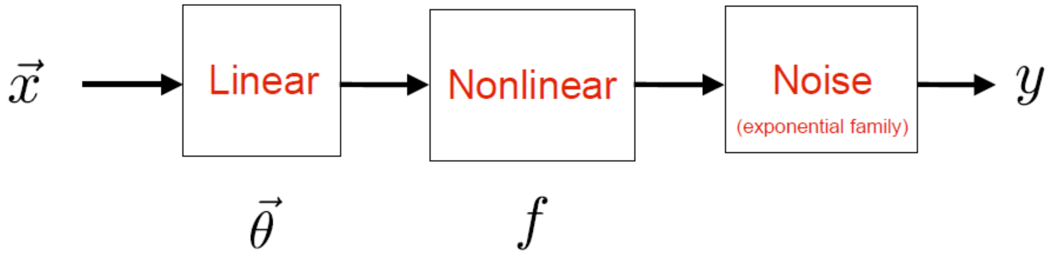


Examples:

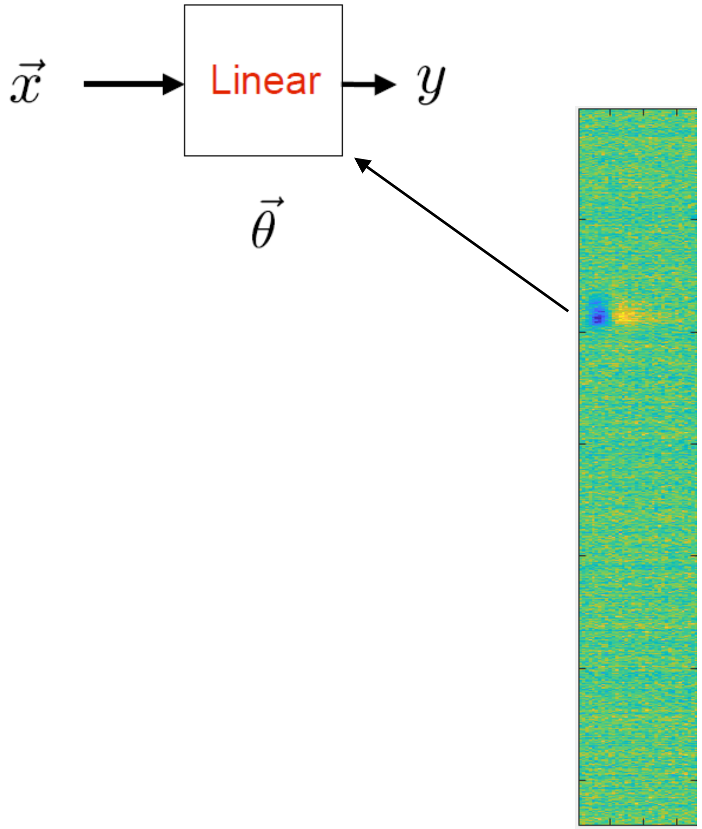
1. Gaussian $y = f(\vec{\theta} \cdot \vec{x}) + \sigma^2 \epsilon$
2. Poisson $y \sim \mathrm{Poiss}(f(\vec{\theta} \cdot \vec{x}))$

How does this compare to the STAs we computed last week?

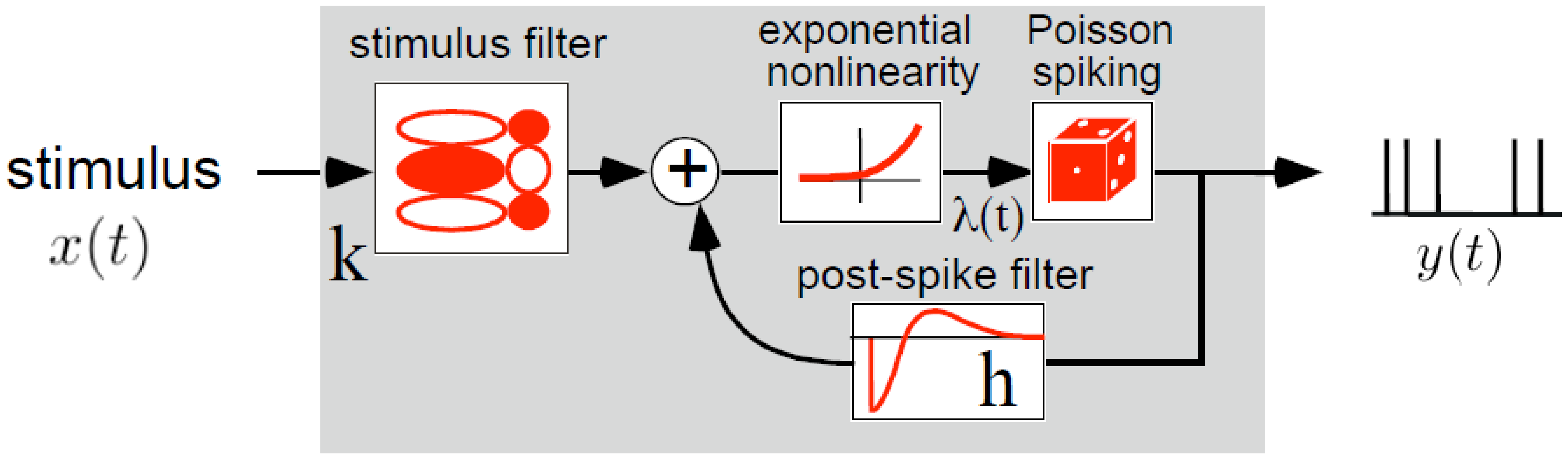
Generalized Linear Model



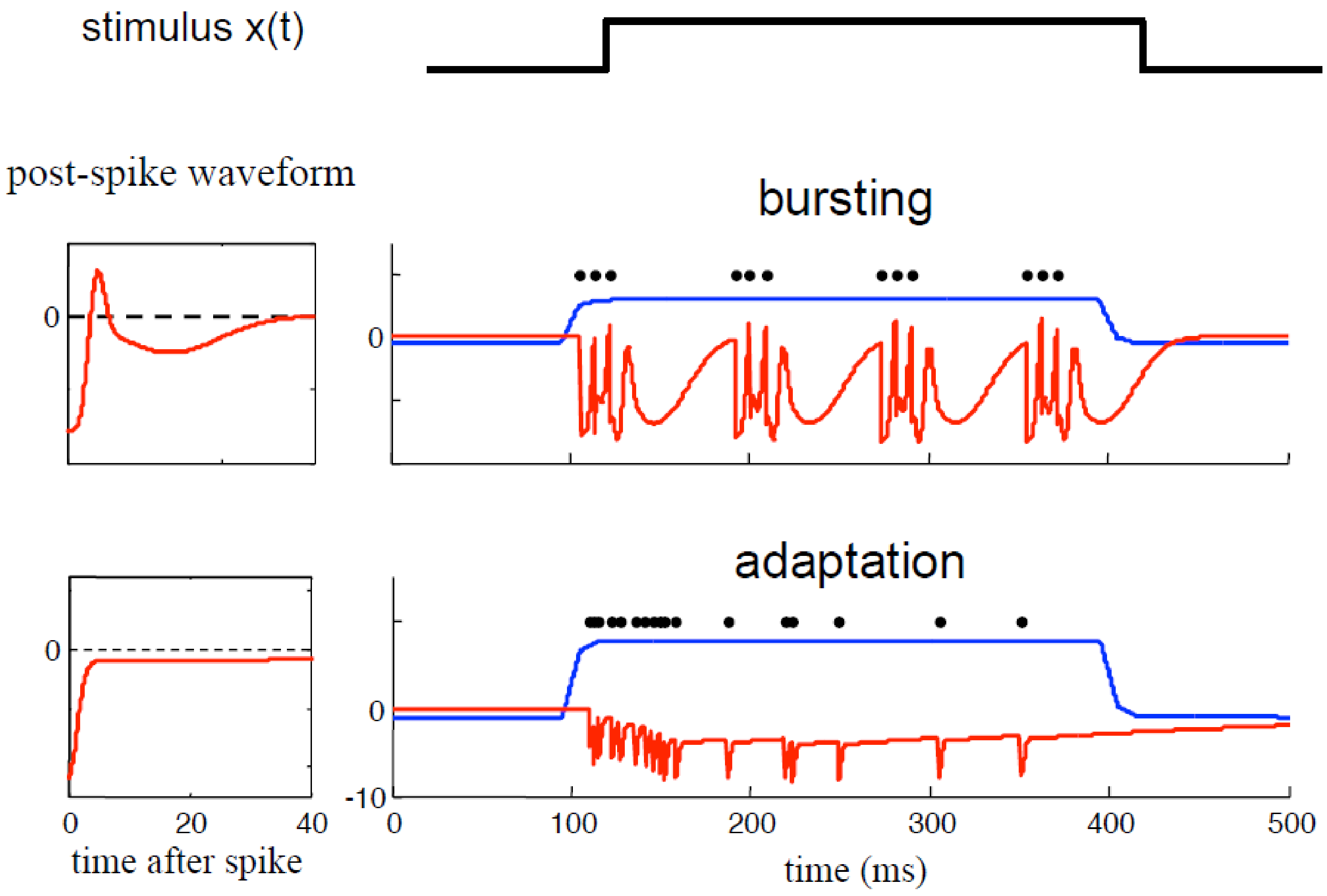
Linear "STA" Model



Adding spike-history effects



Adding spike-history effects



Evaluating your model

```
                Results: Generalized linear model
==============================================================
Model:              GLM              AIC:            211.5952
Link Function:      log              BIC:            -77.9931
Dependent Variable: y                Log-Likelihood: -97.798
Date:               2019-02-01 12:57 LL-Null:        -111.41
No. Observations:   32               Deviance:       5.1846
Df Model:           7                Pearson chi2:   5.14
Df Residuals:       24               Scale:          1.0000
Method:             IRLS
---------------------------------------------------------------
            Coef.    Std.Err.     z      P>|z|    [0.025  0.975]
---------------------------------------------------------------
const       5.7793    1.4768   3.9134  0.0001   2.8849  8.6738
x1         -0.0025    0.0021  -1.1825  0.2370  -0.0066  0.0016
x2         -0.1046    0.0677  -1.5449  0.1224  -0.2372  0.0281
x3          0.0045    0.0035   1.2853  0.1987  -0.0024  0.0114
x4         -0.0069    0.0055  -1.2549  0.2095  -0.0176  0.0039
x5          0.0000    0.0000   0.5213  0.6021  -0.0000  0.0000
x6          0.0310    0.0316   0.9800  0.3271  -0.0310  0.0929
x7          0.0001    0.0001   1.2860  0.1984  -0.0001  0.0003
==============================================================
```